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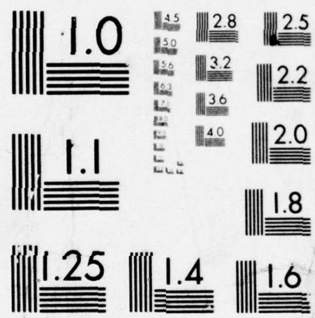


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6 OPTIMAL WAGE RATES AND FORCE COMPOSITION
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# OPTIMAL WAGE RATES AND FORCE COMPOSITION IN MILITARY MANPOWER PLANNING

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## ABSTRACT

A mathematical model of a military manpower system is presented which seeks to determine the optimal steady state wage rate and force distribution by length of service. Accessions and retention are the transition parameters of a steady state manpower model and these are assumed to be a function of wages. A productivity function is introduced to measure effectiveness of each force structure. Gradient search is used to find the compensation plan that will produce a long run force with maximum effectiveness within a given limited annual budget.

## I. INTRODUCTION

The All-Volunteer Force (AVF) has had some obvious effects on the management of military manpower by the services and the Office of the Secretary of Defense. This is particularly obvious in the area of initial procurement where the influence of the AVF has resulted in higher pay for first-termers, in greater recruiting expense, and in changes in the qualitative attributes and motivation of recruits. The fact that first-term personnel are both more expensive and less plentiful has strong implications for the composition and structure of the military enlisted force.

One of the principal issues that manpower planners must face is determining the best, or optimal, structure of the AVF by length of service. The static composition of the force is an extremely important determinant of force cost and force effectiveness. Over a period of years *retention policy* (the proportion of men continuing from one term of service to the next) and the *accession policy* (the number of initial enlistments) together determine this force.

The military compensation system can be made to regulate enlistments and reenlistments. This function has been explicitly recognized, as can be seen in the use of bonuses and proficiency pay to promote accessions and retention in particular occupational areas. Although pay levels are set by Congress and might not be thought of as variable, changes in the structure of the military pay system may be possible as a result of increasing experience with the AVF.* A companion issue to that of military compensation is the system of military retirement. The retirement system also has an impact on the composition and structure of the military force, which is evident from the high retention rates (near 100 percent) for men approaching 20 years of service and the high loss rates among men who have become eligible for retirement pay. The impact on retention and losses of such changes as a partial vesting of retirement benefits must be considered by military manpower planners.

The approach taken in the manpower model presented here is to treat military manpower policies as variables and thus to investigate higher-level policy issues related to military compensation, retention, and the rate of initial accessions. The model can be used to investigate many of the factors that are held constant in other manpower models. This model seeks to determine the *optimal composition* of the military enlisted force by term of service. The optimal force is defined as that force which provides the greatest military capability for a given budget cost.

The cost of hiring personnel is determined by military manpower supply functions which related enlistment and reenlistment rates to military pay. In applying the model, we are able to draw on research which has estimated supply functions for initial enlistment and first-term reenlistment.[3-7,10,11,13-17] The productivity of the force is measured by an index which, in the simplest case analyzed, gives productivity weights to men in each term of service. "Optimal" rates of pay are determined by maximizing this productivity index,

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*Within the past ten years, radical changes have, in fact, occurred in the military compensation system: comparability pay increases (1965-1970); special pay programs, such as the variable reenlistment bonus (1966) and the combat arms enlistment bonus (1972); and the AVF pay increases for first-termers (1969-1971). The first opportunity to consider further large-scale changes in the military pay system, reflecting some experience with the AVF, will be the Third Quadrennial Review of Military Compensation, to be submitted to Congress in 1975.

subject to a budget constraint. These optimal rates of military pay determine the number serving in each term of service under steady-state conditions. The optimization procedure seeks to balance the incremental productivity of a class of personnel with the incremental cost of hiring personnel in that term.

Discrete dynamic programming or Markov decision theory is applicable when the future behavior of a system can be predicted on the basis of the current state, time, and decision but independent of the path (sequence of states, times, and decisions) taken to get to the present. This theory is concerned with the optimal sequential control of a periodically observed and controlled stochastic process.

Many researchers have determined the form of the optimal policy for specific models that are specifically restricted subsets of the general Markov decision process. The construction of optimal policies for these models is often an even more difficult problem. Fortunately, policies can be found using construction algorithms developed by these same researchers for some of these models.

Flynn [(8)] considers a deterministic Markov decision model with direct application to the military manpower system. This system consists of productive units that age and possibly leave the system early or ultimately retire. The production rate is a linear function of the number of units in the different age groups. The general decision on wage and recruitment is allowed to be a function of both time and the current state of the system. An optimal policy in this model is one which minimizes the total present worth of all payments subject to the productivity rate constraint.

Flynn shows that minimizing the average cost as an alternative to the total present worth criterion will produce a good target state (long-run manpower force distribution). This target state is the same stationary point that the optimal policy for the total present worth criterion would ultimately reach, and a "steady-state" policy can be constructed to reach this target state from the initial system state.

Once the target state $x^*$ is reached, $x(p,t)$ never deviates from $x^*$ under the optimal policy ($x^*$ is called a fixed or stationary point of the process). With these results in mind, we need not even consider x as time variant and need only find the best target or stationary x by minimizing an expression called the average cost criterion which differs from the optimal cost by an amount whose bound is independent of the interest rate. Thus, while a truly optimal policy is almost impossible to calculate, a good one can be easily found by minimizing the average cost criterion.

These results can also be used to justify the use of the long-run policy for the dual problem of maximizing the average productivity rate of the linear production model developed in this report subject to a constraint on annual costs. If this maximum productivity is chosen as the value of the productivity constraint imposed by Flynn, then the target state and wage decisions are identical and this average cost criterion will be minimized and will equal our annual budget. Thus the model we present here determines a decision and target state that approaches the optimal decision.

We assume that the total number of men in year of service (YOS) class j ($j=1, 2, \ldots n$) is characterized by variable $x_j$. We define the state description vector $x = (x_j)$. Similarly, pay vector p is used to describe yearly pay $p_j$ perceived by members of YOS j. The state of the system is thus characterized by vector x, which is in general a function of both pay p and time t and is written $x(p,t)$. The total annual personnel budget is constrained to be less than or equal to B dollars, and productivity (military effectiveness) is measured by a function of the number of men in each year of service class and written $S(x)$.

The problem becomes: max $S(x)$, subject to $p'x \leq B$. When written as a total Lagrangian with $\lambda$ expressed in terms of output per dollar, the resulting problem is to find critical points of $L(p,\lambda) = S(x) + \lambda(B - p'x)$ with respect to p and $\lambda$ with all variables nonnegative. Steady-state force composition described by x is a function of p, $x(p)$.

The general solution yields equations of the form

$$\sum_{i=1}^{n} \frac{\partial x_i}{\partial p_j} (s_i - \lambda p_i) - \lambda x_j = 0, \text{ for each } j$$

and

$$\sum_{i=1}^{n} p_i x_i = B, \text{ where } s_i = \frac{\partial S(x)}{\partial x_i} .$$

The critical points of L can be obtained by solving n+1 nonlinear equations for n+1 unknowns. This is generally difficult unless the functions x(p) and S(x) have a particular form allowing for recursive or iterative methods for solution.

## II. THE MODEL: MANPOWER SUPPLY AND PRODUCTIVITY

To make the model of the military personnel system explicit, it is necessary to specify the following: (1) a particular production function, (2) an explicit form for the enlistment and retention functions $r(p)$, and (3) military pay received and the pay "perceived" by enlistees and reenlistees. In making these assumptions we move from the area of applied mathematics into the area of the economics of labor supply and production where we must attempt to develop mathematical forms which embody the most realistic economic and behavioral assumptions. One simplifying assumption we have made is that instead of modelling separate year groups, which would require at least 20 separate seniority classes, we have compressed the time to the four-year increments reflecting the term of enlistment. The twenty-year military career pursued by the enlisted man is thereby modeled by five four-year terms.

In manpower systems such as the military, which have no lateral entry, the supply of manpower for the *ith* term is equal to the number of enlistees in term i-1 times the subsequent retention rate. Thus, for the highest term

$$x_5 = x_1 r_2 r_3 r_4 r_5,$$

and in general

$$x_i = x_1 r_2 r_3 \dots r_i.$$

We define $x_1 = r_1$ to simplify notation. Each supply function is assumed to depend on the annual pay perceived by potential enlistees and reenlistees. Although civilian earnings and tastes for military service obviously influence military manpower supply, these are not variables within the context of the model but rather are included indirectly in parameters of the retention functions. Thus, we assume

$$r_i = r_i(p_i).$$

To represent the supply of enlistees and reenlistees, we have adopted the function

$$r(p) = ce^{-ap^{-b}} ,$$

where the parameters a, b, c would differ for each term of service. This exponential function is a positively sloped s-shaped function and is bounded by 0 and c for all positive levels of perceived pay. (Examples plotting r(p) for several sets of parameter values are shown in Fig. 1.) This guarantees, for instance, that reenlistment rates will always fall between 0 and c regardless of the level of military pay. The parameter c for the reenlistment functions $r_1, \ldots, r_5$ serves two purposes: It reflects the proportion of men in the previous term eligible to reenlist, and for the special case of $r_1$, which represents the number of enlistees rather than an enlistment rate, $c_1$ reflects the total pool of men eligible to enlist. The attractiveness of the exponential function lies in both its mathematical tractability and the reasonableness of its shape. The derivative of reenlistments with respect to perceived pay is

$$\frac{dr}{dp} = abp^{-b-1} r.$$

This formulation greatly simplifies the difficult problem of finding a solution.

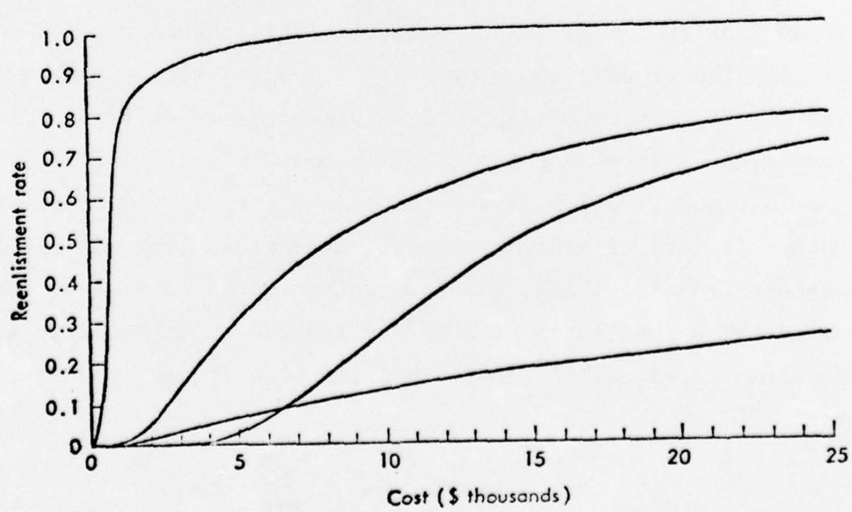


Fig. 1 — Retention rates as a function of pay for various a and b parameter values ($r(p) = \exp(-a.p^{-b})$)

The military pay perceived by an individual considering enlisting or reenlisting in the military will almost certainly not be limited to the pay rate applying at the time of enlistment or reenlistment. Our formulation of the military pay perceived by the prospective enlistee or reenlistee is nearly unique in that it includes military pay beyond the term under consideration.

The military retirement system, which provides an annuity for men who have completed twenty years of service, is a major reason for high retention rates in the career force. Since retirement requires a minimum of twenty years of service, the retirement pay is actually "earned" during the fifth four-year term because men who serve only four terms receive no retirement payments under the present system.

To reflect future pay, such as retirement, in the enlistment and reenlistment decision, we construct an expected present value of military earnings from each term of service onward. Expected present value of earnings is the sum of present and future earnings weighted by a discount factor and the probability of remaining in the military, which can be calculated from the retention rate $r_1, \ldots, r_5$. Hence, the present value at term of service i is of the form $w_i + r_{i+1}d_{i+1}w_{i+1} + r_{i+1}r_{i+2}d_{i+1}d_{i+2}w_{i+2} + \ldots$. The present value is then put on an annualized basis by dividing by the appropriate annuity factor, calculated as the sum of the weights used in the expected present value calculation. This single measure in dollars per year is called perceived pay and is the sole determinant through the retention function r of reenlistment percentages. This annual rate is weighted average of the annual wage rates, $w_1, w_2, \ldots, w_5$. (Retirement is included in $w_5$.) The model, as written, is general enough to allow different discount rates for different points in the military career, $d_2, d_3, d_4, d_5$.

In this model, which is devoted to the composition of military manpower, all other factors of production, such as various types of capital equipment and categories of civilian labor, are treated as constants. We investigate two production functions: the linear and the multiplicative (Cobb-Douglas) production functions. In vector notation the linear production function is written

$$S(x) = s'x = \sum_{j=1}^{n} s_j x_j .$$

Here the contributions of the different year classes are additive, and the elasticity of substitution between different year classes is infinite. In a sense, the different year classes are perfect substitutes for one another, in that tradeoffs (even unlimited) can be made between $x_i$ and $x_j$ at the ratio $s_j/s_i$ without any sacrifice in productivity.

In the Cobb-Douglas production function

$$S(x) = x_1^{\alpha_1} x_2^{\alpha_2} \dots x_5^{\alpha_5}.$$

The sum of the parameters $\Sigma a_i$ is a measure of the returns to scale and is arbitrarily set equal to 1.0, since results of our model are independent of this normalization. Furthermore, in this function the elasticity of substitution is 1.0. In the Cobb-Douglas some quantity of input from each labor class is required to produce any level of output.

## III. THE SOLUTION

Assume vector s defines the productivity per man in each term of service. A distinction has been described between perceived pay p and wage, which are related through the linear transformation $p = Aw$. The problem is, maximize x's subject to $x'w \le B$, with

$$x(p) = \begin{pmatrix} r_1(p_1) \\ r_1(p_1)\ r_2(p_2) \\ r_1(p_1)\ r_2(p_2)\ r_3(p_3) \\ \cdot \\ r_1(p_1)\ \cdots\cdots\ r_5(p_5) \end{pmatrix} .$$

Since p is the decision variable driving state vector x, we can transform the problem to one involving p alone: maximize x's subject to $x'A^{-1}p \le B$, where

$$A^{-1} = \begin{bmatrix} 1 + r_2d_2 + r_2r_3d_2d_3 \cdots + r_2d_2 \cdots r_5d_5 & - \cdots & 0 & 0 & 0 \\ 0 & 1 + \cdots & - r_3d_3 - r_3r_4d_3d_4 - r_3d_3r_4d_4r_5d_5 & 0 & 0 \\ 0 & 0 & 1 + r_4d_4 + r_4r_5d_4d_5 & - r_4d_4 - r_4r_5d_4d_5 & 0 \\ 0 & 0 & 0 & 1 + r_5d_5 & - r_5d_5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} .$$

The total Lagrangian is $L = s'x - \lambda[x'A^{-1}p - B]$.

The term $x'A^{-1}p$ is a scalar and when expanded can be reordered. This reordering indicates that the same scalar could be constructed from $x'Cp$, where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ d_2 & 1 - d_2 & 0 & 0 & 0 \\ d_2d_3 & d_3(1 - d_2) & 1 - d_3 & 0 & 0 \\ d_2d_3d_4 & d_3d_4(1 - d_2) & d_4(1 - d_3) & 1 - d_4 & 0 \\ d_2d_3d_4d_5 & d_3d_4d_5(1 - d_2) & d_4d_5(1 - d_3) & d_5(1 - d_4) & 1 - d_5 \end{bmatrix}.$$

Taking the derivatives of L with respect to p to obtain the first-order conditions for critical points is easier using this matrix, C, which is independent of p.

$$\frac{\partial L}{\partial p} = \frac{\partial x'}{\partial p} \cdot [s - \lambda Cp] - \lambda C'x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\frac{\partial L}{\partial \lambda} = B - x'Cp = 0.$$

We define $M \equiv \frac{\partial x'}{\partial p}$ .

In our model

$$M = \begin{Bmatrix} a_1b_1p_1^{-b_1-1} \cdot (r_1, r_1r_2, r_1r_2r_3, \ldots, r_1r_2r_3r_4r_5) \\ a_2b_2p_2^{-b_2-1} \cdot (0, r_1r_2, r_1r_2r_3, \;.\,, \quad . \quad ) \\ \vdots \qquad\qquad \vdots \\ a_5b_5p_5^{-b_5-1} \cdot (0, \quad 0, \quad 0, \quad 0, r_1r_2r_3r_4r_5) \end{Bmatrix}.$$

For the case of the Cobb-Douglas production function, Flynn's analysis may not apply. Theoretical work is needed before we could state that our steady-state approach has produced a good policy in the same sense. The steady-state optimal policy for the Cobb-Douglas production function can be found by maximizing the revised Lagrangian

$$L = \sum_{i=1}^{5} \alpha_i \ln x_i - \lambda[x'A^{-1} p-B].$$

The first-order conditions are

$$\frac{\partial L}{\partial p} = - \lambda MCp - \lambda C'x + \left\{ \begin{matrix} a_1 b_1 p_1^{-b-1} \cdot \sum_1^5 \alpha_i \\ a_2 b_2 p_2^{-b-1} \cdot \sum_2^5 \alpha_i \\ \cdot \\ \cdot \\ \cdot \\ a_5 b_5 p_5^{-b_5-1} \cdot \alpha_5 \end{matrix} \right\}.$$

Gradient search methods are used in either model to find a near-zero gradient. Lagrange multiplier step size and perceived pay step size are varied separately as a function of the number of interactions and of the decrease in the length of the gradient vector. Convergence to a local optimum occurs generally within 40 iterations but is very critically dependent upon the step size selection procedure. Optimal step size algorithms are not used but could be implemented if necessary. No attempt has been made to prove or test empirically that this local is global optimum, but our opinion is that this is a global extreme point.

Experience has indicated that convergence of the gradient search may not occur if the step size selection is not satisfactory. Computation time on The Rand Corporation IBM 370/158 took less than 12 seconds for compilation and execution.*

The initial starting point in any gradient method is important since the search may move toward the multiple inflection, maximum, or

*A program listing is available from the authors.

minimum points of the nonlinear Lagrangian. The starting point used here is found by using a perceived budget constraint rather than the correct direct expenditure budget. Hopefully, this will allow us a starting point close to the global optimum. In the optimization procedure, maximize $s'x$ with a perceived budget constraint $p'x \leq B$. Differentiation of the total Lagrangian with respect to basic decision variables $p_j$ produces the following equations for each j:

$$a_j b_j p_j^{-b_j-1} \left[ s_j - \lambda p_j + \sum_{i=j+1}^{n} r_i(p_i) \cdot r_{i-1}(p_{i-1}) \cdots r_{j+1}(p_{j+1})(s_i - \lambda p_i) \right] - \lambda = 0 .$$

These are a set of n nonlinear equations of degree $b_j + 1$ and can be solved recursively given each $\lambda$. When solved, $p_n^*$ and $r_n(p_n^*)$ can be substituted into the (n-1) equation. This recursion continues so that each equation contains only one decision variable and can be solved numerically using Newton's method given a specific value for the Lagrange multiplier $\lambda$.

Everett's(2) results assure us that if we happen to find a $\lambda$ that achieves the budget constraint, then we should use it and the policy generated by i, since it is optimal (or near optimal). Fox and Landi(9) review methods for finding $\lambda$ in the case of one constraint and suggest bisection which proceeds by successively halving an interval $\lambda_1$ and $\lambda_2$. The two starting points are chosen so that the budget generated $B(\lambda_1) < B$. When the midpoint $\lambda_k$ of the *kth* iteration produces a budget $B(\lambda_k)$ sufficiently close to B, the procedure stops. Bisection is mini-max in the sense that it minimizes the maximum number of iterations required to locate the root in an interval of fixed length. A subroutine has been written using bisection to converge to a $\lambda^*$ such that $B(\lambda^*)$ is within one percent of the desired perceived budget B to obtain a good starting solution for the main program.

## IV. NUMERICAL RESULTS

The principal source of existing data for developing parameter values for the model is the considerable econometric research into the supply of military personnel. Production function S(x) was estimated using preliminary estimates obtained from an on-the-job training study under way at Rand.[12] For the linear production model, this provided us with an estimate that first-term average productivity was 0.68 compared with productivity of fully trained journeymen. During the second term, the average productivity was assumed to be 1.0. Subsequent productivity was obtained by viewing civilian alternative wage rates of workers in similar fields and background by number of years on the job. Becker[1] has provided rough estimates of civilian wage increased by age, which when converted to term of service were 1.28, 1.513, and 1.755.

Discount factors for first term were constructed using an interest rate of 30 percent per year ($d_2 = (1/1.30)^4 = 0.350$). During the second term, the time value of money was calculated using 10 percent per year ($d_3 = 0.683$). Discount for subsequent terms and for retirement is based on 5 percent per year ($d_4$, $d_5 = 0.823$). Precedence for using these values for terms one and two can be found in Refs. 1 and 4.

Productivity, current wages, current retention, and perceived pay generated using the appropriate discount factors and retention rates are shown in Table 2. The table also shows the optimal values of wage, retention, and perceived pay for the linear production functions; similar results were obtained using a Cobb-Douglas production function.

Currently first-termers represent 55 percent of the USAF enlisted force with an enlistment rate of 76,500 per year. In the optimal schedule for linear production this figure is reduced to 38 percent with enlistments of 56,800 required annually. Total productivity increased 16 percent, and total number of men in service increased by 7.35 percent. These changes would leave the annual budget unchanged at $5.174 billion.

The slightly irregular pattern of military pay reported under the optimal results is not as troublesome as it may appear. In fact, military pay in any term can be thought of as the sum of regular compensation, bonuses, and retirement vesting. Under the present system, of course, reenlistment bonuses occur mainly in the second term and retirement vesting occurs at the twenty-year point or the fifth four-year term, creating considerable irregularity.

Table 2

Current and Optimal Pay Rates and Manpower Distribution

| Item | Term of Service 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| **Current (Transient) Manpower (1972)** | | | | | | |
| Average wage, $ | 6,110 | 8,130 | 8,040 | 8,630 | 22,030[a] | $5.174 billion |
| Perceived pay, $ | 6,840 | 10,210 | 11,870 | 14,600 | 22,030 | |
| Gross retention[b] | 0.20 | 0.273 | 0.765 | 0.951 | 0.973 | |
| Number of men | 306,100 | 68,600 | 47,200 | 45,000 | 89,800 | 556,700 |
| Productivity | 0.68 | 1.0 | 1.28 | 1.513 | 1.755 | 562,400 units |
| **Optimal Steady State Manpower Plan Using Linear Production Function** | | | | | | |
| Average wage, $ | 3,100 | 11,300 | 10,940 | 10,700 | 16,300 | $5.174 billion |
| Perceived pay, $ | 5,590 | 11,760 | 12,140 | 13,060 | 16,300 | |
| Gross retention | 0.137 | 0.518 | 0.785 | 0.910 | 0.888 | |
| Number of men | 227,800 | 118,000 | 92,600 | 84,300 | 74,900 | 597,600 |
| Productivity | 0.68 | 1.0 | 1.28 | 1.513 | 1.755 | 650,100 units |

[a]Wage in the fifth term consists of current $9,230 average wage plus $12,800 annual equivalent for the retirement component.

[b]Gross retention = $x_{j+1}/x_j$ uncorrected with $c_j$.

Table 3 indicates an example of a wage, bonus, and retirement vesting program that achieves this optimal pay schedule while providing a smooth and increasing stream of regular compensation. More study is needed to determine the best combination.

Table 3

**WAGES, BONUS, AND RETIREMENT**

**(In dollars)**

| Term | Optimal Wages | Annual Basic Pay | Reenlistment Bonus | Annual Retirement Vesting |
|---|---|---|---|---|
| 1 | 3,100 | 3,100 | 0 | 0 |
| 2 | 11,300 | 6,000 | 4,000 | 1,300 |
| 3 | 10,940 | 8,000 | 1,500 | 1,440 |
| 4 | 10,700 | 9,000 | 0 | 1,700 |
| 5 | 16,300 | 10,000 | 0 | 6,300 |

**NOTE:** All figures are in annual equivalents.

By way of analysis of the results so far obtained, it does appear that the higher first-term reenlistments combined with lower career force reenlistment rates as a whole are desirable. The effect of learning the job, as indicated by low productivity during the first term, is the primary factor. With greater retention, a reduction in the number of enlistees required to maintain the staffing levels and output is feasible.

Sensitivity analysis of the results of Table 2 to changes in productivity and supply reenlistment parameters was conducted. While actual optimal wage and retention rates varied as expected, the qualitative conclusions persisted that a decrease in the number of first-terms (accomplished by reducing their pay) and an increase in retention of these first-termers (accomplished by increasing pay) were both desirable.

Furthermore, retention at the end of the third and fourth terms was somewhat reduced by slight reduction in pay. We postulated in Table 3 that an early retirement vesting system could be useful to (a) smooth out the optimal wage fluctuations required to achieve optimal retention and (b) make early-out options more equitable and appealing. Further study of this proposal and its effect on retention seems advisable.

## V. CONCLUSIONS

At this point in the study, conclusions for extensive policy changes are obviously premature. The purpose of this research has been to demonstrate the usefulness of the approach and verify the model. Further work must be undertaken to refine the rough estimates of retention parameters and productivity before recommendations would be warranted. Also, as we have indicated, it would be desirable to reimpose on this system some of the present legal and institutional constraints. This will permit us to examine the cost of such constraints.

Our model can be applied to subdivisions of the service such as a single military occupation specialty. Here productivity and reenlistment and supply parameters might more easily be measured.

Once a steady-state optimal policy and target states are known, the next question is how to achieve them. This problem has not been addressed here. Preliminary work suggests that a transient policy might be found using dynamic or mathematical programming and furthermore that the policy is highly dependent upon the interest rate. We are able to construct good policies, in the same sense that Flynn[8] describes them. This type of transient policy simply guarantees reaching the optimal target state in a finite number of steps (in our case, n periods), not necessarily in an optimal way.

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